Easy Logarithm Problems

1. Given $\log_a 16 = 2$, find **(a)** a **(b)** $\log_{16}\left(\frac{1}{a}\right)$.

(a) $\log_a 16 = 2 \Rightarrow a^2 = 16 = 4^2 \Rightarrow a = 4$

(b) $\log_{16}\left(\frac{1}{a}\right) = \log_{16}(a)^{-1} = -\log_{16}a = -\frac{1}{\log_a 16} = -\frac{\mathbf{1}}{\mathbf{2}}$

2. Given $p = \frac{1}{q^4}$, find (a) $\log_q p$ (b) $2\log_p q$

(**a**) **Method 1**

$p = \frac{1}{q^4} \Rightarrow \log p = \log\frac{1}{q^4} = \log 1 - \log q^4 \Rightarrow \log p = -4\log q$

$\log_q p = \frac{\log p}{\log q} = \frac{-4\log q}{\log q} = -\mathbf{4}$

Method 2

$p = \frac{1}{q^4} \Rightarrow \log_q p = \log_q \frac{1}{q^4} = \log_q 1 - \log_q q^4 = 0 - 4\log_q q = -\mathbf{4}$

(b) $2\log_p q = \frac{2}{\log_q p} = \frac{2}{-4} = -\frac{\mathbf{1}}{\mathbf{2}}$

3. If $5^m = 7^n = 35^p$, express p in terms of m and n.

Method 1

$5^m = 7^n = 35^p \Rightarrow m\log 5 = n\log 7 = p\log 35 = k$

$\log 5 = \frac{k}{m}$, $\log 7 = \frac{k}{n}$, $p = \frac{k}{\log 35} = \frac{k}{\log 5 + \log 7} = \frac{k}{\frac{k}{m}+\frac{k}{n}} = \frac{1}{\frac{1}{m}+\frac{1}{n}} = \frac{mn}{m+n}$

Method 2

$5^m = 35^p \Rightarrow 5 = 35^{\frac{p}{m}}$ ... (1)

$7^n = 35^p \Rightarrow 7 = 35^{\frac{p}{n}}$ ... (2)

$(1) \times (2)$, $35 = 35^{\frac{p}{m}+\frac{p}{n}} \Rightarrow \frac{p}{m} + \frac{p}{n} = 1 \Rightarrow p = \frac{mn}{m+n}$

4. Given $log_x 9 = y$, find $log_9 81x$ in terms of y.

$log_x 9 = \frac{1}{log_9 x} = y, \quad log_9 x = \frac{1}{y}$

$log_9 (81x) = log_9 81 + log_9 x = log_9 9^2 + log_9 x = 2 + \frac{1}{y}$

5. If $5^m = 7^n = 35p$, express p in terms of both m and n.

$5^m = 35p,\ 5^m = 5(7p), 5^{m-1} = 7p,\ p = \frac{5^{m-1}}{7}$

$7^n = 35p,\ 7^n = 7(5p),\ 7^{n-1} = 5p,\ p = \frac{7^{n-1}}{5}$

$p^2 = \left(\frac{5^{m-1}}{7}\right)\left(\frac{7^{n-1}}{5}\right), p = \pm\sqrt{5^{m-2}7^{n-2}}$

6. Given $h = 3^x$ and $k = 3^y$.

(a) Express $\frac{27^{x+y}}{9^x}$ in terms of h and k.

(b) Express $log_9 \frac{9h^2}{k}$ in terms of x and y.

(a) $\frac{27^{x+y}}{9^x} = \frac{3^{3(x+y)}}{3^{2x}} = \frac{(3^x)^3(3^y)^3}{(3^x)^2} = \frac{h^3k^3}{h^2} = hk^3$

(b) $log_9 \frac{9h^2}{k} = log_9 9 + 2log_9 h - log_9 k = 1 + 2log_9 3^x - log_9 3^y = 1 + log_9 3^{2x} - \frac{1}{2}log_9(3^y)^2$

$= 1 + xlog_9 9 - \frac{1}{2}ylog_9 9 = 1 + x - \frac{y}{2}$

7. **(a)** Simplify $log_3(4p+1) - 3log_9 p^2 + 4log_3 p$

(b) Solve $log_3(4p+1) - 3log_9 p^2 + 4log_3 p = 1$

(a) $log_3(4p+1) - 3log_9 p^2 + 4log_3 p = log_3(4p+1) - 3\frac{log_3 p^2}{log_3 9} + 4log_3 p$

$= log_3(4p+1) - 3\frac{log_3 p^2}{log_3 3^2} + 4log_3 p = log_3(4p+1) - 3\frac{log_3 p^2}{2log_3 3} + 4log_3 p$

$= log_3(4p+1) - \frac{3}{2}log_3 p^2 + 4log_3 p = log_3(4p+1) - 3log_3 p + 4log_3 p$

$= log_3(4p+1) + log_3 p = log_3 p(4p+1)$

(b) $log_3(4p+1) - 3log_9p^2 + 4log_3p = 1$

$log_3p(4p+1) = 1$

$p(4p+1) = 3^1$

$4p^2 + p - 3 = 0$

$\therefore p = -1 \ or \ p = \frac{3}{4}$

$p = -1$ is rejected since it cannot satisfy the given equation, therefore $\boldsymbol{p = \frac{3}{4}}$.

8. Show that $\mathrm{a}^{\log_c b} = \mathrm{b}^{\log_c a}$

Since $\log_c\left(\mathrm{a}^{\log_c b}\right) = \log_c b \log_c a = \log_c a \log_c b = \log_c\left(\mathrm{b}^{\log_c a}\right)$

Therefore $\mathrm{a}^{\log_c b} = \mathrm{b}^{\log_c a}$

9. Given $\log_a b = \log_b c = \log_c a$, show that a = b = c .

Let $\log_a b = \log_b c = \log_c a = u$

$a^u = b, b^u = c, c^u = a \ \ldots (1)$

$a = c^u = (b^u)^u = b^{(u^2)} = (a^u)^{(u^2)} = a^{u^3}$

$\therefore u^3 = 1$, $u^3 - 1 = 0$, $(u-1)\,(u^2+u+1) = 0$

$\therefore u = 1$ (u is real)

From (1), a = b = c .

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